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Impacts of Investment Horizon on the Estimation of Beta Coefficients, Jensen Measure and Efficient Frontier: Lognormal vs. Normal Distribution Approach

Cheng F. Lee

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Impacts of Investment Horizon on the Estimation of Beta Coefficient, Jensen Measure and Efficient Frontier: Lognormal vs. Normal Distribution Approach

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Abstract

Based upon the lognormal distribution assumption the impacts of investment horizon on the estimation of beta coefficient, Jensen measure and efficient frontier are analyzed in detail. It is found that investment horizon is one of the most important factors in estimating beta coefficient, Jensen measure and efficient frontier. It is also shown that the standard Jensen measure estimate is biased. An unbiased Jensen measure is derived in accordance with the property of intercept for a log-linear regression. Empirical results are used to support the analytical results derived in this paper.

I. Introduction

Portfolio theory developed by Markowitz (1957) and Tobin (1958, 1965) is essentially based upon the assumptions that portfolio's rates of return are normally distributed and the investor's utility function is quadratic. Recently Elton and Gruber (1974) have developed a portfolio theory in terms of lognormally distributed investment relatives. Gressis, Philippatos and Hayya (1976) have shown that the investment horizon will generally affect the portfolio analysis. In addition, Levy (1972), and Lee (1976A, 1976B) and Levhari and Levy (1977) have analyzed the impact of investment horizon on the systematic risk estimates and the investment performance measure estimates. However, these analyses have not explicitly taken the problem associated with alternative distribution assumptions into account.

The main purpose of this paper is to analyze the possible impacts of statistical distribution and investment horizon on the estimates of beta coefficient, Jensen performance measure, and efficient frontier parameter estimates of a portfolio. Sharpe's (1963) diagonal model is used to examine how the investment horizon can affect the composition of an efficient portfolio. In the second section, a lognormal capital asset pricing model (CAPM) with investment horizon parameter is derived in accordance with the results developed by Lee (1976) and Bawa and Chakrin (1979); some portfolio theories on the efficient portfolio determination under a lognormal market are reviewed in accordance with the results developed by Elton and Gruber (1974), Ohlson and Ziemba (1976) and Levy (1973). In the third section, the regression for the bivariate lognormal distribution is discussed in accordance with

Heien (1968) and Goldberger (1968). The implications of these results on the Jensen investment performance measure in terms of investment horizon will be explored in some detail. In the fourth section the impacts of investment horizon on the beta estimates and the performance measure estimates under both normal distribution and lognormal distribution capital asset pricing model will be explored and the implications of these results to the efficient portfolio in terms of the Sharpe's (1963) diagonal model will be examined. Data of 45 firms randomly selected from the New York Stock Exchange (NYSE) will be used to do some empirical studies. To study the bias of Jensen measure, 464 firms will be used to do the empirical analysis. Finally, the results of this paper will be summarized and some concluding remarks will be indicated.

II. Investment Horizon and the Lognormal CAPM

In a paper examining the relationship between investment horizon and the systematic risk estimate, Lee (1976B) has derived a lognormal CAPM in terms of a statistical concept. Bawa and Chakrin (1979) [BC] have also derived a lognormal CAPM in terms of portfolio analysis identical to that derived by Lee (1976B) and drawn some possible implications to the risk-return relationship test. However, BC do not consider the problem associated with investment horizon.

Following Lee (1976B) and Jensen (1969), the risk-return relationship for the capital asset pricing model can be defined as

$$(1) \qquad E({}_H R_j) = {}_H R_f (1 - {}_H\beta_j) + E({}_H R_m)\ {}_H\beta_j,$$

where H = the true investment horizon, ${}_{H}R_{j}$ = the rate of return on security j, ${}_{H}R_{m}$ = the market rate of return, ${}_{H}R_{f}$ = the risk-free rate of interest, and ${}_{H}\beta_{j}$ = the system risk.

Equation (1) implies the risk return tradeoff is linear when the true investment horizon is known. However, the true investment horizon generally is unknown. If the observed horizon is defined as N, then the relationship between ${}_{H}R_{j}$ and ${}_{N}R_{j}$, and ${}_{H}R_{m}$, and ${}_{N}R_{m}$, and ${}_{H}R_{f}$ and ${}_{N}R_{f}$ can be defined as:

$$(2) \qquad {}_{H}R_{i}^{*} = (1 + {}_{H}R_{i}) = (1 + {}_{N}R_{i})^{\lambda} = {}_{N}R_{i}^{*\lambda} \qquad (i = j, m, f)$$

where $\lambda = H/N$, after substituting (2) into (1), we have

$$(3) \qquad [E({}_{N}R_{j}^{*})]^{\lambda} = ({}_{N}R_{f}^{*})^{\lambda} (1-{}_{H}\beta_{j}) + [E({}_{N}R_{m}^{*})]^{\lambda} ({}_{H}\beta_{j}),$$

Equation (3) can be rewritten as:

$$(4) \qquad E({}_{N}R_{j}^{*}) = [{}_{N}R_{f}^{*\lambda} (1-{}_{H}\beta_{j}) + [E({}_{N}R_{m}^{*})]^{\lambda} {}_{H}\beta_{j}]^{1/\lambda} .$$

If the observed rates of return, ${}_{N}R_{j}^{*}$ and ${}_{N}R_{m}^{*}$ are log normally distributed, following Aitchison and Brown (1957), we have

$$(5) \qquad E({}_{N}R_{m}^{*}) = \exp(\sigma_{1}^{2}/2 + \mu_{1}) \qquad (a)$$

$$\text{var}({}_{N}R_{m}^{*}) = \exp(2\mu_{1} + \sigma_{1}^{2}) [\exp(\sigma_{1}^{2})-1] \qquad (b)$$

where $\mu_{1} = E(\log {}_{N}R_{m}^{*})$ and $\sigma_{1}^{2} = \text{var}(\log {}_{N}R_{m}^{*})$.

To apply the Cramer's rule to (5), we obtain

$$(6) \qquad E(\log {}_{N}R_{m}^{*}) = \log[\mu^{2}R_{m}^{*}/\sqrt{\sigma^{2}R_{m}^{*} + \mu^{2}R_{m}^{*}}] \qquad (a)$$

$$\text{var}(\log {}_{N}R_{m}) = \log[E({}_{N}R_{m}^{*2})] - \log[E^{2}(R_{m}^{*})] \qquad (b) .$$

For deriving $\text{cov}[(\log {}_N R_j^*), (\log {}_N R_m^*)]$, we let $r_1 = \log {}_N R_j^*$ and $r_2 = \log {}_N R_m^*$ be bivariate normally distributed with mean vector

$$\mu = (\mu_1, \mu_2) \text{ and covariance matrix } W = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = V^{-1}.$$

The bivariate distribution of $R = (r_1, r_2)'$ can be written as[1]

$$(7) \qquad g(r_1, r_2) = \frac{1}{2\pi |w|^{1/2}} \exp\{-1/2[(R-\mu)'V(R-\mu)]\}.$$

Using the definition of covariance and $g(r_1, r_2)$, it can be shown that[2]

$$(8) \quad \text{cov}[{}_N R_j^*, {}_N R_m^*] = \exp(\mu_1 + \mu_2)\{\exp[1/2(\sigma_1^2+\sigma_2^2+2\sigma_{12})] - \exp[1/2(\sigma_1^2+\sigma_2^2)]\}.$$

From (8), it can also be shown that

$$(9) \qquad \sigma_{12} = \log[(\text{cov}({}_N R_j^*, {}_N R_m^*)/\exp(\mu_1 + \mu_2)\exp(1/2(\sigma_1^2+\sigma_2^2))) + 1].$$

After substituting equations (6a) and (6b) into (9), we have

$$(10) \qquad \sigma_{12} = \log\{E[({}_N R_j^*)({}_N R_m^*)]\} - \log[E({}_N R_j^*)\, E({}_N R_m^*)].$$

From (5b) and (8), the finite and the systematic risk can be defined as[3]

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[1]Derivation of the density function for a bivariate normal distribution can be found in Hogg and Craig (1969).

[2]The derivation of this result is available from the author.

[3]Following Jensen (1969) and Cheng and Deets (1973), the finite systematic risk can be defined as: ${}_N\beta_j = \text{cov}({}_N R_j^*, {}_N R_m^*)/\text{var}({}_N R_m^*)$.

$$(11) \qquad {}_N\beta_j = C_j \left\{ \frac{\exp[1/2(\sigma_1^2+\sigma_2^2+2\sigma_{12})] - \exp[1/2(\sigma_1^2+\sigma_2^2)]}{[\exp(\sigma_1^2)-1]} \right\}$$

where $C_j = \exp(\mu_1 + \mu_2)/\exp(2\mu_1 + \sigma_1^2)$.

Using the definition lognormal distribution, we have

$$(12) \qquad E({}_N R_m^*) = \exp(\sigma_1^2/2 + \mu_1) \qquad (a)$$

$$E({}_N R_j^*) = \exp(\sigma_2^2/2 + \mu_2) \qquad (b)$$

from equations (11) and (12), we have

$$(13) \qquad {}_N\beta_j = (E({}_N R_j^*)/E({}_N R_m^*))\ [(\exp(\sigma_{12}) - 1)/(\exp(\sigma_1^2) - 1)]$$

Substituting equation (13) and $\lambda = 1$ into equation (4), we have

$$(14) \qquad \frac{E({}_H R_j^*) - {}_H R_f^*}{E({}_H R_j^*)} = \left(\frac{\exp(\sigma_{12}) - 1}{\exp(\sigma_1^2) - 1}\right) \left[\frac{E({}_H R_m^*) - {}_H R_f^*}{E({}_H R_m^*)}\right]$$

Bawa and Chakrin (1979) have used equation (14) to explore the possible implications of the lognormal CAPM on the risk-return relationship trade-off test.

If λ is a nonnegative rational number, then both ${}_N R_j^{*\lambda}$ and ${}_N R_m^{*\lambda}$ are lognormal, hence

$$(15) \qquad E({}_N R_m^{*\lambda}) = \exp[\lambda^2\sigma_1^2/2 + \lambda\mu_1] \qquad (a)$$

$$\mathrm{var}({}_N R_m^{*\lambda}) = \exp(2\lambda\mu_1 + \lambda^2\sigma_1^2)\ [\exp(\lambda^2\sigma_1^2) - 1] \qquad (b)$$

$$E({}_N R_j^{*\lambda}) = \exp[\lambda^2\sigma_2^2/2 + \lambda\mu_2] \qquad (c)$$

$$\mathrm{var}[{}_N R_j^{*\lambda}) = \exp(2\lambda\mu_2 + \lambda^2\sigma_2^2)\ [\exp(\lambda^2\sigma_2^2) - 1] \qquad (d)$$

Following the same procedure used in this section, we can obtain

$$(16) \qquad {}_N\beta_j(\lambda) = [E({}_NR_j^{*\lambda})/E({}_NR_m^{*\lambda})]\ [\exp(\lambda^2\sigma_{12}) - 1/\exp(\lambda^2\sigma_1{}^2) - 1]$$

${}_N\beta_j(\lambda)$ can be used to investigate the possible relationship between the estimated systematic risk and the investment horizon. This issue will be explored in section IV of this paper.

III. Jensen Performance Measure Under Bivariate Lognormal CAPM

Lee (1976A) has derived an approximate CAPM for Equation (4) in terms of logarithm transformation as

$$(17) \qquad \log {}_NR_j^* - \log {}_NR_f^* = \alpha_j + {}_H\beta_j' (\log {}_NR_m^* - \log {}_NR_f^*) + {}_H\gamma_j (\log {}_NR_m^* - \log {}_NR_f^*)^2 + \varepsilon_j$$

where ${}_H\gamma_j = 1/2\lambda\ {}_H\beta_j\ (1-{}_H\beta_j)$ and ε_j is the disturbance term. This specification is subject to specification bias unless λ is smaller than one. Nevertheless, the estimated ${}_H\gamma_j$ of (17) can generally be used to test whether λ is significantly different from zero. If λ is larger than one, the estimated ${}_H\gamma_j$ will be affected by the omitted higher order terms except that all omitted terms are not correlated with the quadratic excess market rate of return. This is one source of specification error. The model defined in equation (17) has been used by Treynor and Mazuy (1966) and Jensen (1972) in determining the unbiased measure of stock selection ability. Even the standard specification as defined in equation (14) is correct, there still exists some problem in estimating so-called Jensen performance measure in terms of a bivariate log normal regression as defined in Equation (18).

(18) $$\log(Z_1) = \alpha_j + \beta_j \log(Z_2) + \varepsilon_{jt}$$

where $Z_1 = {}_N R^*_{jt} / {}_N R^*_{ft}$ and $Z_2 = {}_N R^*_{mt} / {}_N R^*_{ft}$, ε_{jt} is normally distributed with mean zero and variance σ^2_ε.

Based upon Heien (1968) and Goldberger (1968), the log-linear regression as indicated in equation (18) can be written as

(19) $$Z_1 = \gamma_j Z_2^{\beta_j} \exp(\varepsilon_j)$$

where γ_j = anti log α_j; ε_j is normally distributed with mean zero and variance σ^2_j. Under this assumption, it is clear that

(20) $$E(Z_1) = \gamma_j Z_2^{\beta_j} \exp(1/2\ \sigma^2_{\varepsilon_j})$$

Both Goldberger (1968) and Heien (1968) have shown that the ordinary least square (OLS) estimate of α_j should be defined as[4]

(21) $$\hat{\alpha}'_j = (\overline{\log Z_1} - \hat{\beta}_j \overline{\log Z_2}) - 1/2\ \sigma^2_{\varepsilon_j} = \alpha_j - 1/2\ \sigma^2_{\varepsilon_j}$$

where $\overline{\log Z_1}$ is average excess rates of return for jth security and $\overline{\log Z_2}$ is average excess market rates of return, therefore, the first item of RHS in equation (21) is the so-called Jensen investment performance measure plus the residual variance of jth security. Therefore, the unbiased estimated Jensen measure should be defined as

(22) Unbiased Jensen Measure (UJM) = $\hat{\alpha}_j + 1/2\ \sigma^2_{\varepsilon j}$

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[4] See Appendix A.

Equations (21) and (22) imply that the traditional Jensen measure estimate is not independent of the non-systematic risk of an individual security or a portfolio. In other words, $\hat{\alpha}_j$ biased against a security (or portfolio) with larger residual variance (non-systematic risk). Most recently, Professor Roll (1978) has shown that the Jensen measure is not an unambiguity measure for investment performance. His conclusion is essentially based upon the problem of index measurement. Our results here are based upon the assumption that the market index is free from the measurement problem.

Merton (1970) and Jensen (1972, 385-386) have used the lognormal and the discrete interval assumption to derive an equilibrium CAPM and draw a similar conclusion to those derived in this section. In sum, Merton and Jensen concluded that the theoretical intercept of CAPM is $- 1/2 [\sigma_j^2 - \beta_j \sigma_m^2]$. If Merton and Jensen's results are used, then the unbiased Jensen measure can be defined as

(22') Unbiased Jensen measure = $\hat{\alpha}_j + 1/2[\sigma_j^2 - \beta_j \sigma_m^2]$

The difference between Merton and Jensen's results and the result derived in this paper is to be studied in the future research.

IV. Investment Horizon, Systematic Risk, Jensen Measure and Efficient Portfolio

Cheng and Deets (1973), Levhari and Levy (1976) and Lee (1976A, 1976B) have shown that investment horizon does have some impact on the estimates of systematic risk; Levy (1972) and Levhari and Levy (1977) have demonstrated that both Sharpe investment performance measure

and Treynor performance measure can be affected by the investment horizon. By using the Markowitz (1959) model, Gressis, Philippatos and Hayya (1976) have shown that the efficient portfolio determination is not independent of the investment horizon. However, the lognormal distribution assumption has not been explicitly integrated with investment horizon to investigate the impact of investment horizon on the estimated systematic risk and the estimated investment performance measures.

The relationship between the estimated systematic risk and investment horizon under a lognormal assumption can be analyzed in accordance with the results indicated in equation (16). There exist two components, i.e., $[E({}_N R_j^{*\lambda})/E({}_N R_m^{*\lambda})]$ and $[\exp(\lambda^2\sigma_{12}) - 1/\exp(\lambda^2\sigma_1{}^2) - 1]$ to affect the change of ${}_N\beta_j(\lambda)$. If $\lambda = 1$, then equation (16) reduces to equation (13). If $\lambda \neq 1$, then the change of ${}_N\beta_j(\lambda)$ with respect to the change of λ can be analyzed as follows.

From equations (15a) and (15c), we have

$$(23) \qquad \frac{E({}_N R_j^{*\lambda})}{E({}_N R_m^{*\lambda})} = \left[\frac{e^{\lambda^2\sigma_1{}^2+\lambda\mu_1}}{e^{\lambda^2\sigma_2{}^2+\lambda\mu_2}}\right] = e^{\lambda^2(\sigma_1{}^2-\sigma_2{}^2)+(\mu_1-\mu_2)\lambda}$$

Taking derivative of $\frac{E({}_N R_j^{*\lambda})}{E({}_N R_m^{*\lambda})}$ with respect to λ, then

$$(24) \qquad \frac{\partial}{\partial\lambda}\left[\frac{E({}_N R_j^{*\lambda})}{E({}_N R_m^{*\lambda})}\right] = e^{\lambda^2(\sigma_1^{22}-\sigma_2{}^2)+(\mu_1-\mu_2)\lambda}\left[\lambda(\sigma_1{}^2-\sigma_2{}^2)+(\mu_1-\mu_2)\right]$$

In equation (24), the first item is always larger than zero, hence, the sign is determined by the second term. From the second term of this equation, it can be shown that

(i) when λ increases, $\left[\frac{E({}_N R_j^{*\lambda})}{E({}_N R_m^{*\lambda})}\right]$ will increase if $\lambda > \frac{(\mu_2-\mu_1)}{\sigma_1^2-\sigma_2^2}$

(ii) when λ increases, $\left[\frac{E({}_N R_j^{*\lambda})}{E({}_N R_m^{*\lambda})}\right]$ will reduce if $\lambda < \frac{(\mu_2-\mu_1)}{\sigma_1^2-\sigma_2^2}$

Now, the relationship between $\left[\frac{e^{\lambda^2\sigma_{12}}-1}{e^{\lambda^2\sigma_1^2}-1}\right]$ and λ is explored. It can be shown that

$$(25)\qquad \frac{\partial\left[\frac{(e^{\lambda^2\sigma_{12}}-1)}{(e^{\lambda^2\sigma_1^2}-1)}\right]}{\partial\lambda} = \frac{[2\lambda\sigma_{12}e^{\lambda^2\sigma_{12}}][e^{\lambda^2\sigma_1^2}-1]-[2\lambda\sigma_1^2 e^{\lambda^2\sigma_1^2}][e^{\lambda^2\sigma}}{(e^{\lambda^2\sigma_1^2}-1)^2}$$

The numerator of equation (24) can be simplified as

$$(26)\qquad 2\lambda[\sigma_{12}(a-b) - \sigma_1^2(a-c)]$$

where $a = e^{\lambda^2(\sigma_{12}+\sigma_1^2)}$

$b = e^{\lambda^2\sigma_1^2}$

$c = e^{\lambda^2\sigma_{12}}$

The sign of equation (26) is jointly determined by λ^2, σ_{12} and σ_1^2 and it is a non-linear function of these variables. Therefore, the sign of equation (25) can hardly be determined. If the magnitude of both $\lambda^2\sigma_{12}$ and $\lambda^2\sigma_1^2$ are relatively small, it can be shown that

$(e^{\lambda^2\sigma_{12}}-1)/(e^{\lambda^2\sigma_1^2}-1)$ is approximately equal to $\frac{\sigma_{12}}{\sigma_1^2}$.[5] Under this circumstance, the impact of λ on ${}_N\beta(\lambda)$ is essentially determined by the term $[E({}_NR_j^{*\lambda})/E({}_NR_m^{*\lambda})]$. From equation (24), it has been shown that the relationship between λ and $(\mu_2-\mu_1)/(\sigma_1^2-\sigma_2^2)$ is the key factor in determining the change of beta coefficient as the observation unit change. If the magnitude of $\lambda^2\sigma_1^2$ and $\lambda^2\sigma_{12}$ is not negligible, then the relationship between λ and ${}_N\beta(\lambda)$ becomes ambiguous.[6]

To investigate the impact of investment horizon on the efficient portfolio, 45 securities as listed in Appendix B are randomly selected from the New York Stock Exchange (NYSE). The sample period is during January, 1966-December, 1979. Sharpe's (1963) diagonal model as defined in equation (26) is used to formulate the efficient portfolio for 12 alternative investment horizons.

$$(26) \qquad {}_NR_{jt} = \alpha_j + \beta_j {}_NR_{mt} + \varepsilon_{jt}$$

where ${}_NR_{jt}$ = rates of return for jth security in period t

${}_NR_{mt}$ = market rates of return in period t

α_j and β_j are intercept and slope for jth security. Note that N represents the observation horizon for R_{jt} and R_{mt}. In this empirical study, the N represents either one month, two months, ..., or twelve months. The information required for Sharpe's diagonal model is ${}_N\hat{\alpha}_j$, ${}_N\bar{\beta}_j$, ${}_N\sigma_j^2$ and ${}_N\sigma_m^2$. The compositions of all possible efficient portfolios

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[5] See Appendix B for the derivation.

[6] This is due to the fact that the higher order terms are not negligible.

under twelve different investment horizons are listed in Appendix C. Appendix C indicates that number of possible efficient portfolio is not independent of the investment horizons used to construct the efficient frontier. These results also indicate that the estimated parameters for efficient portfolio are not independent of the different observation horizons used (see Table I).[8] The main reason for this difference is due to the fact that the estimated beta coefficients are not independent of observation horizons. The results of estimated beta for 12 different investment horizons are listed in Table II. These findings are similar to those found by Cheng and Deets (1973), Levhari and Levy (1977) and others. These results have supported the results as indicated in equation (16).

To investigate how the bias associated with the estimated OLS Jensen measure, data of 464 firms during the periods of 1966-72 and 1972-79 are used to estimate the bias Jensen measure.[7] Fisher index is used to calculate the market rates of return and the monthly 90 day treasury bill rate is used as the proxy of risk-free rate. Both the standard Jensen measure and the unbiased Jensen measures as defined in equations (22) and (22') are calculated. The summary results of these three different kinds of Jensen measures are listed in Table IV. In table III, $\bar{\alpha}_1$, $\bar{\alpha}_2$, and $\bar{\alpha}_3$ are average biased and unbiased estimated Jensen measures respectively. By comparing $\bar{\alpha}_1$ with $\bar{\alpha}_2$ and $\bar{\alpha}_3$, it can be concluded that

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[7] 45 firms used in investigating the impacts of investment horizon on the composition of efficient frontier is randomly selected for these 464 firms.

[8] The portfolios listed in Table I is one of possible set of efficient portfolios listed in Appendix C.

Table I

Estimated Parameters of Efficient Portfolios

Number of Periods, N	$\mu(k)$	$\sigma(k)$	Coefficient of variation (σ_k/μ_k)	Number of securities included in the portfolio
1	.00030	.03241	108.03	13
2	.00312	.04544	14.56	17
3	.00357	.05871	16.45	11
4	.00849	.05404	6.37	14
5	.01040	.06504	6.25	13
6	.01270	.07375	4.76	12
7	.01549	.07095	4.58	14
8	.01998	.06620	3.31	13
9	.00629	.07643	12.15	11
10	.05316	.07625	1.43	16
11	.01170	.09079	7.75	8
12	.04140	.08693	2.10	16

Table II

Beta Coefficients for 12 Investment Horizons

horizons / firm no.	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m
1	1.10822	1.60350	1.61631	1.47018	1.73555	1.93290	1.54034	1.86171	1.55612	1.90435	2.81247	1.67447
2	1.70420	2.46162	2.07980	2.66479	2.26629	1.77623	2.23273	2.08694	1.50137	2.46768	2.72158	1.85971
3	0.89651	1.26628	0.97488	0.72093	0.91418	0.72581	0.65882	0.64588	0.93226	0.46044	0.46835	0.18217
4	1.76065	0.92693	1.03458	1.02775	0.86295	0.74049	1.05344	0.79214	1.56960	0.86874	0.54297	0.81975
5	0.96433	1.47750	1.49063	1.54476	1.35416	1.81945	1.53583	1.81693	0.98419	1.51467	1.85510	1.60966
6	1.23751	1.18657	1.19379	1.26237	1.20859	1.21282	1.34361	0.94938	2.20455	0.71970	0.38736	0.78013
7	1.64269	0.88016	0.80115	0.89970	0.69619	0.92079	0.89719	1.04094	0.94550	0.94640	0.85810	1.13247
8	2.15099	2.02691	1.84191	2.00590	2.13116	2.09976	2.89991	1.84234	1.99730	2.32177	1.19779	1.49842
9	1.16388	0.98242	0.81385	0.97479	0.72322	0.80905	0.86395	1.11189	0.68025	0.95795	0.69445	1.00719
10	1.08813	1.81746	1.84115	1.68566	1.94893	2.11142	1.92541	2.11749	1.51115	1.77142	1.62523	2.06538
11	1.36439	0.93101	0.79005	0.58822	0.54827	0.38700	0.51859	0.07586	0.75503	0.38337	0.45853	0.14075
12	1.17769	1.05716	0.99332	1.02471	1.07079	1.04870	0.63412	1.25702	0.20604	0.06837	1.00553	1.08329
13	0.83714	0.82197	0.75787	0.82033	0.90045	1.01389	0.74382	0.64874	1.34009	0.47403	0.95264	0.56314
14	1.16581	1.02923	0.98893	0.98581	1.00276	0.97284	0.79471	0.90625	0.62699	0.90117	0.76337	0.82586
15	0.87819	0.76509	0.70823	0.46981	0.48579	0.86912	0.74327	0.75554	0.88948	0.40225	0.91845	0.59139
16	1.36313	1.39782	1.10492	1.15522	1.24926	1.03547	0.87545	0.86177	0.35353	0.79851	0.70679	0.88974
17	1.01200	1.10480	1.02936	1.39322	1.16615	1.33786	1.34478	1.23200	1.17887	1.33132	0.91806	1.33482
18	1.20219	1.20564	1.14846	1.19482	1.11084	1.30345	0.88064	1.62291	0.41791	1.01925	1.52028	1.27304
19	1.53456	1.60523	1.65751	1.63172	1.82915	1.85392	1.84203	1.91025	1.55188	1.89805	1.85393	1.84091
20	1.27471	1.21903	1.40086	1.16856	1.29399	1.44020	1.22159	1.41547	0.81466	1.19462	1.49286	1.16209
21	0.96670	0.57970	0.65933	0.70967	0.69885	0.33330	0.20523	-0.06022	0.42175	-0.17343	0.97874	-0.18051
22	1.18136	1.51381	1.49321	1.45186	1.25608	1.54922	1.70112	1.35763	1.56335	1.87101	1.90890	1.57783
23	1.75441	2.05987	2.17066	2.34827	1.34753	2.45794	2.21296	3.03107	1.37316	2.48313	3.13408	6.51798
24	0.99424	0.90119	1.06278	0.67893	0.73376	0.63396	0.63451	0.54227	0.78452	0.21709	0.90500	0.53653
25	1.19494	1.17578	1.04935	0.82102	0.92910	0.94506	0.60704	0.93786	0.54161	0.55263	0.68102	0.88943
26	1.31564	1.34491	1.40747	1.64248	1.54666	1.82404	2.23038	2.25036	1.73947	2.22057	1.84093	2.45873
27	1.01203	0.56588	0.73013	0.50420	0.63970	0.67556	0.41019	0.80453	0.29795	0.67219	0.67607	0.71922
28	1.04565	1.42301	1.34964	1.47608	1.45250	1.48545	1.06350	1.25354	0.83688	0.94145	1.50935	1.13583
29	0.67679	0.56637	0.45950	0.35610	0.34999	0.13396	0.47119	0.29302	0.19498	0.32723	0.16880	0.24297
30	0.87598	0.96080	0.94376	0.76911	0.57494	1.16729	0.49697	0.96094	0.46277	0.64784	0.67456	1.15096
31	1.30518	1.32153	1.25611	0.97646	0.42437	1.14857	1.19013	0.96722	1.24269	1.25781	1.12024	1.22445
32	0.66297	0.50176	0.56630	0.25250	0.53335	0.61614	0.20017	0.23619	0.75644	0.17764	0.67588	0.09546
33	0.61890	0.53752	0.59024	0.55693	0.59732	0.62173	0.45213	0.74760	0.47118	0.75341	0.45525	0.73058
34	0.45250	0.66505	0.66543	0.53967	0.56880	0.45735	0.45414	0.49162	0.32855	0.61563	0.49424	0.53419

Table II
(continued)

horizons / firm no.	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m
35	0.71571	0.63566	0.65720	0.70491	0.54486	0.66167	0.60406	0.49869	0.65367	0.69208	0.69747	0.60118
36	1.42786	1.48467	1.85022	2.01127	1.97246	1.90721	3.02125	2.12615	2.14669	2.31515	2.38684	2.11906
37	0.76857	0.75819	0.84125	0.89933	0.98846	0.97364	1.02452	0.97355	0.86390	1.10072	0.44622	1.09919
38	1.22452	1.34781	1.17813	1.44637	1.25600	1.22191	1.30145	1.21972	0.93080	1.27586	1.24451	1.18268
39	1.03628	1.19815	1.15433	0.80285	0.84030	0.99146	0.77241	0.62874	0.49677	0.44248	1.43597	0.64804
40	0.95770	0.94154	0.94296	0.53354	0.81547	0.55294	0.65048	0.46035	0.91941	0.58729	0.58936	0.63446
41	0.85077	0.75548	0.89905	0.33133	0.22150	0.51072	0.32122	0.46247	0.94771	0.20225	0.76228	0.68528
42	0.61666	0.70936	0.68472	0.74523	0.76676	0.81857	0.94307	0.95708	0.79372	1.12193	0.60069	1.08828
43	1.07147	1.45041	1.34178	1.45751	1.46528	1.63517	1.35690	1.77245	1.12543	1.51180	1.70808	1.83316
44	1.53019	2.11104	1.79185	1.80392	1.82646	2.08700	1.50010	1.83973	0.86062	1.89923	2.05758	1.84552
45	0.68402	0.62311	0.45044	0.35491	0.40193	0.53031	0.30179	0.60988	0.32579	0.39152	0.89659	0.52335

Remarks: 1) Name of the firms can be found in Appendix B.
2) Horizons are one month, two months, ..., and twelve months.

Table III

	$\bar{\alpha}_1$	s_1	$\bar{\alpha}_2$	s_2	$\bar{\alpha}_3$	s_3
1966-72	-.00223	.00738	.00042	.00808	.00151	.00839
1973-79	.00093	.00898	.00422	.01060	.00573	.01105

α_1 = Jensen Performance Measure

$\alpha_2 = \alpha_1 + 1/2\sigma_{\varepsilon j}^2$

$\alpha_3 = \alpha_1 + 1/2[\sigma_j^2 - \beta_j \sigma_m^2]$

the standard OLS Jensen measure estimate is an under-estimated Jensen measure. Therefore, the standard Jensen measure estimate does not fairly evaluate the performance of mutual fund, portfolio and individual securities. To study the possible ranking bias of using standard biased Jensen estimates instead of unbiased Jensen measure estimates. Both ranking correlation and product-moment correlation are used to check the ranking differences caused by using $\hat{\alpha}_1$ instead of $\hat{\alpha}_2$ and $\hat{\alpha}_3$. It is found that all correlation coefficients are higher than .945. These results imply that the standard Jensen measure estimate can still be used to rank the performance of mutual fund, portfolios and individual securities.

V. Summary

In this paper, the empirical relationships between investment horizon and portfolio analysis are analyzed in accordance with both normal and lognormal assumptions. Especially, the lognormal distribution is used to investigate possible impact of investment horizon on the estimated beta in terms of an analytical functional relationship. In addition, the possible bias of estimated Jensen measure in terms of a bivariate log linear specification is analyzed. Finally, some empirical efficient portfolio estimates in terms of different investment horizons are used to show that investment horizon is an important factor in portfolio analysis. It is also shown that the standard Jensen measure does exhibit estimation bias.

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APPENDIX A

If the regression of equation (18) is a normal linear regression with the assumption that each e_{jt} is $N(0,\sigma_\varepsilon^2)$. From the distribution theory it is known that if ε_j is normal then $e^{\varepsilon j}$ is log-normal. Under this circumstance, the expected Z_1 as indicated in (20) is not equal to $\gamma_j Z_j \beta_j$. To make the expected disturbance for the multiplicative specification as indicated in equation (19) equal to one, equation (19) can be reparameterized as

(i) $$Z_1' = \gamma_j Z_2 \exp(\varepsilon_j - 1/2\sigma_\varepsilon^2$$

Equation (i) implies that

$$E(Z_1') = \gamma_i Z_2 \; E[\text{Exp}(\varepsilon_j - 1/2\sigma_\varepsilon^2)]$$

$$= \gamma_j Z_2$$

Taking the log transform of equation (i), we have

(ii) $$\log Z_1' = \alpha_j + \beta_j \log Z_2 + \mu_i$$

where $$\mu_j = \log \varepsilon_j - 1/2\sigma_\varepsilon^2$$

In equation (ii), μ_i is $N(-1/2\sigma_\varepsilon^2, \sigma_\varepsilon^2)$. If we applied the ordinary least square (OLS) estimation method to equation (ii), we actually estimate the specification of equation (iii) instead of (ii).

(iii) $$\log Z_1' = \alpha_j' + \beta_j \log Z_2 + \log \varepsilon_j$$

where $\alpha_j' = \alpha_j - 1/2\sigma_{\varepsilon j}^2 = (\overline{\log Z_1} - \hat{\beta}_j \overline{\log Z_2})$

This is equation (21). Further discussion on the issues discussed here can be found in Heien (68) and Goldberger (1968).

APPENDIX B

$$e^{\lambda^2\sigma_{12}} - 1 = \lambda^2\sigma_{12} + \frac{1}{2!}(\lambda^2\sigma_{12})^2 + \frac{1}{3!}(\lambda^2\sigma_{12})^3 + \ldots \qquad \text{(a)}$$

$$e^{\lambda^2\sigma_1{}^2} - 1 = \lambda^2\sigma_1{}^2 + \frac{1}{2!}(\lambda^2\sigma_1{}^2)^2 + \frac{1}{3!}(\lambda^2\sigma_1{}^2)^3 + \ldots \qquad \text{(b)}$$

If $\lambda^2\sigma_{12}$ and $\lambda^2\sigma_1{}^2$ approach zero, then the higher order terms are negligible, and therefore

$$\frac{e^{\lambda^2\sigma_{12}} - 1}{e^{\lambda^2\sigma_1{}^2} - 1} = \frac{\sigma_{12}}{\sigma_1{}^2} .$$

APPENDIX B

Sample List for 45 Firms

1.	NWT	Northwest Inds Inc
2.	TWA	Trans World Airls Inc
3.	PBI	Pitney Bowes Inc
4.	WLA	Warner Lambert Co
5.	SMC	Smith A O Corp
6.	JOY	Joy Mfg Co
7.	BGG	Briggs & Stratton Corp
8.	TF	Twentieth Centy Fox Film Corp
9.	FIR	Firestone Tire & Rubr Co
10.	PVH	Phillips Van Heusen Corp
11.	PFE	Pfizer Inc
12.	GR	Goorich B F Co
13.	CLL	Continental Oil Co
14.	ACF	A C F Inds Inc
15.	UCL	Union Oil Co Calif
16.	HML	Hammermill Paper Co
17.	AD	Amsted Inds Inc
18.	RVS	Reeves Bros Inc
19.	TXT	Textron Inc
20.	NG	National Gypsum Co
21.	IGL	International Minerals & Chem
22.	DG	Associated Dry Goods Corp
23.	REP	Republic Corp
24.	DE	Deere & Co
25.	JWL	Jewel Cos Inc
26.	CK	Collins & Aikman Corp
27.	SUO	Sheel Oil Co
28.	MLR	Midland Ross Corp
29.	CCC	Continental Group Inc
30.	RHR	Rohr Inds Inc
31.	HFC	Household Fin Corp
32.	SOH	Standard Oil Co Ohio
33.	LOU	Louisville Gas & Elec Co
34.	PAC	Pacific Tel & Teleg Co
35.	NFK	Norfolk & Westn Ry Co
36.	RCC	Royal Crown Cola Co
37.	RGS	Rochester Gas & Elec Corp
38.	JCP	Penney J C Inc
39.	TU	Trans UN Corp
40.	CIT	C I T Final Corp
41.	ENS	Enserch Corp
42.	NMK	Niagara Mahawk Pwr Corp
43.	NAS	National Svc Inds Inc
44.	AIC	American Invt Co
45.	SUN	Sun Inc

APPENDIX C

Compositions of Possible Efficient Frontier Twelve Efficient Portfolio

possible efficient portfolios \ horizons	1		2		3		4		5		6	
	$E(R_j)$	σ_j	$E(R_j)$	σ_j	$E(R_j)$	σ_j	$E(R_j)$	σ_j	$E(R_j)$	σ_j	$E(R_j)$	σ_j
1	0.01335	0.13800	0.02922	0.21939	0.05278	0.45199	0.06138	0.49690	0.08699	0.38721	0.14696	0.85052
2	0.01319	0.10166	0.02836	0.16551	0.04975	0.29899	0.06028	0.35876	0.08405	0.34439	0.12342	0.50408
3	0.01303	0.09341	0.02737	0.13429	0.04721	0.22554	0.05842	0.22404	0.08094	0.30452	0.11882	0.44965
4	0.01253	0.07625	0.02604	0.11100	0.04476	0.18488	0.05736	0.20099	0.07158	0.20533	0.10636	0.33118
5	0.01201	0.06642	0.02431	0.09143	0.04150	0.15178	0.05328	0.14707	0.06641	0.16872	0.10105	0.29548
6	0.01201	0.06641	0.02385	0.08800	0.03761	0.12337	0.05173	0.13647	0.06311	0.15284	0.08345	0.19867
7	0.01147	0.06099	0.02345	0.08531	0.03739	0.12214	0.04903	0.12285	0.05484	0.12315	0.07575	0.17044
8	0.01064	0.05468	0.02118	0.07289	0.03419	0.10637	0.04747	0.11660	0.05163	0.11413	0.07511	0.16848
9	0.01059	0.05438	0.02028	0.06933	0.03315	0.10198	0.04447	0.10572	0.05150	0.11378	0.06915	0.15062
10	0.01019	0.05215	0.02028	0.06932	0.03025	0.09110	0.04383	0.10357	0.04974	0.10922	0.06455	0.13767
11	0.01005	0.05141	0.01929	0.06582	0.02982	0.09867	0.04156	0.09648	0.04897	0.10727	0.06229	0.13162
12	0.00900	0.04668	0.01853	0.06340	0.02795	0.08367	0.04145	0.09615	0.04325	0.09355	0.04845	0.09959
13	0.00881	0.04595	0.01762	0.06080	0.02658	0.07969	0.04123	0.09551	0.04230	0.09141	0.04723	0.09734
14	0.00846	0.04474	0.01719	0.05968	0.02550	0.07687	0.03643	0.08248	0.03196	0.07355	0.04447	0.09273
15	0.00839	0.04451	0.01593	0.05674	0.02531	0.07640	0.03538	0.07994	0.03091	0.07246	0.04158	0.08876
16	0.00807	0.04357	0.01542	0.05568	0.02448	0.07445	0.03208	0.07277	0.03037	0.07196	0.04058	0.08761
17	0.00756	0.04224	0.01482	0.05458	0.02231	0.07033	0.03186	0.07235	0.02904	0.07091	0.04000	0.08699
18	0.00682	0.04049	0.01437	0.05385	0.02066	0.06819	0.02742	0.06503	0.02864	0.07064	0.03797	0.08502
19	0.00598	0.03873	0.01236	0.05113	0.02036	0.06787	0.02716	0.06469	0.02800	0.07024	0.03605	0.08342
20	0.00573	0.03758	0.01173	0.05043	0.02001	0.06752	0.02704	0.06453	0.02716	0.06976	0.03278	0.08100
21	0.00446	0.03607	0.01140	0.05008	0.01986	0.06736	0.02296	0.06027	0.02540	0.06885	0.02958	0.07889
22	0.00434	0.03588	0.01061	0.04930	0.01890	0.06642	0.02262	0.06002	0.01372	0.06514	0.02274	0.07556
23	0.00382	0.03513	0.00991	0.04867	0.01468	0.06288	0.02190	0.05948	0.01040	0.06504	0.02063	0.07488
24	0.00283	0.03392	0.00955	0.04836	0.01210	0.06125	0.01473	0.05536			0.01927	0.07452
25	0.00260	0.03368	0.00887	0.04782	0.00507	0.05881	0.01238	0.05456			0.01700	0.07408
26	0.00140	0.03275	0.00775	0.04704	0.00494	0.05879	0.01208	0.05448			0.01270	0.07375
27	0.00124	0.03266	0.00449	0.04561	0.00357	0.05871	0.01184	0.05443				
28	0.00076	0.03247	0.00399	0.04551			0.00849	0.05409				
29	0.00076	0.03247	0.00312	0.04544								
30	0.00066	0.03245										
31	0.00030	0.03241										

APPENDIX C (cont.)

possible efficient portfolios \ horizons	7		8		9		10		11		12	
	$E(R_j)$	σ_j	$E(R_j)$	σ_j	$E(R_j)$	σ_j	$E(R_j)$	σ_j	$E(R_j)$	σ_j	$E(R_j)$	σ_j
1	0.13417	0.57010	0.18505	1.06050	0.21985	1.20948	0.21141	1.08538	0.32596	1.57459	0.77304	3.61953
2	0.12821	0.49479	0.16256	0.67168	0.19424	0.67586	0.20086	0.78237	0.25316	0.68808	0.29348	0.78775
3	0.12212	0.42687	0.14369	0.43863	0.18020	0.51046	0.18323	0.45092	0.24447	0.63890	0.28931	0.76811
4	0.10530	0.27213	0.13574	0.36700	0.17997	0.50883	0.16945	0.35168	0.20008	0.41166	0.27233	0.68888
5	0.09976	0.23347	0.11263	0.20702	0.14709	0.31570	0.16622	0.33231	0.16419	0.27276	0.20749	0.40331
6	0.09110	0.18687	0.10779	0.18615	0.14030	0.29122	0.13320	0.17053	0.15211	0.23945	0.14645	0.19112
7	0.08343	0.15846	0.09523	0.14509	0.13190	0.26371	0.12014	0.13868	0.14712	0.22782	0.13949	0.17590
8	0.07913	0.14649	0.09387	0.14197	0.11871	0.22500	0.11889	0.13650	0.14036	0.21297	0.13871	0.17434
9	0.07302	0.13133	0.08611	0.12531	0.10509	0.18985	0.11801	0.13500	0.13190	0.19588	0.13546	0.16801
10	0.07092	0.12653	0.08007	0.11385	0.09629	0.16912	0.11130	0.12427	0.13075	0.19373	0.12505	0.14921
11	0.07071	0.12607	0.07988	0.11351	0.09584	0.16829	0.10991	0.12219	0.11238	0.16232	0.11734	0.13664
12	0.06631	0.11711	0.07819	0.11056	0.07373	0.12306	0.10961	0.12177	0.11183	0.16146	0.11237	0.12915
13	0.06610	0.11674	0.06696	0.09308	0.07110	0.11857	0.09909	0.10730	0.09988	0.14387	0.10669	0.12131
14	0.06041	0.10691	0.06478	0.09017	0.06612	0.11100	0.09897	0.10715	0.07948	0.12046	0.10005	0.11320
15	0.06009	0.10640	0.06249	0.08733	0.06478	0.10918	0.09755	0.10536	0.07199	0.11462	0.09797	0.11090
16	0.05363	0.09673	0.05959	0.08404	0.05993	0.10340	0.08979	0.09621	0.06071	0.10748	0.09549	0.10830
17	0.05243	0.09509	0.05645	0.08093	0.04819	0.10164	0.08340	0.08959	0.06070	0.10747	0.08592	0.09972
18	0.04893	0.09057	0.05620	0.08070	0.05500	0.09872	0.08176	0.08806	0.05979	0.10695	0.08343	0.09791
19	0.04723	0.08853	0.05261	0.07771	0.05147	0.09584	0.07339	0.08167	0.05121	0.10231	0.08156	0.09668
20	0.03940	0.08056	0.05235	0.07751	0.03776	0.08666	0.07092	0.08030	0.05119	0.10230	0.08112	0.09641
21	0.03731	0.07891	0.04797	0.07456	0.03270	0.08392	0.06817	0.07906	0.04960	0.10150	0.08065	0.09612
22	0.03670	0.07848	0.04647	0.07374	0.03232	0.08373	0.06648	0.07844	0.04533	0.09948	0.07887	0.09511
23	0.03343	0.07636	0.04595	0.07348	0.02491	0.08044	0.05823	0.07655	0.04003	0.09719	0.07872	0.09502
24	0.03329	0.07627	0.04073	0.07104	0.02316	0.07978	0.05578	0.07634	0.01671	0.09105	0.07803	0.09467
25	0.03114	0.07504	0.03112	0.06769	0.01620	0.07767	0.05416	0.07627	0.01230	0.09079	0.07506	0.09340
26	0.02728	0.07324	0.03032	0.06748	0.01471	0.07733	0.05316	0.07625	0.01170	0.09079	0.06937	0.09147
27	0.02727	0.07324	0.01998	0.06620	0.00629	0.07643					0.06776	0.09099
28	0.02622	0.07284									0.04621	0.08708
29	0.02609	0.07280									0.04621	0.08705
30	0.02203	0.07164									0.04140	0.08693
31	0.01728	0.07100										
32	0.01549	0.07095										

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